[Time:3:00 Hrs.] **[Marks: 80]**

Please check whether you have got the right question paper.

N.B:
1. **All questions are compulsory.**
2. **Figures to the right indicate full marks.**
3. **Scientific calculator can be used.**

Q.1	**a)**	Define the norm of a vector in $\mathbb{R}^n$ and prove its properties.	**10**
	b)	Attempt **any Two** of the following:	**10**
	i)	Prove that every bounded sequence in $\mathbb{R}^n$ has a convergent subsequence.	**5**
	ii)	Define the standard topology on $\mathbb{R}^n$ and describe open and closed subsets with examples.	**5**
	iii)	State and prove Bolzano-Weierstrass theorem for sequences in $\mathbb{R}^n$.	**5**
Q.2	**a)**	State and prove the Mean Value Theorem for functions of several variables.	**10**
	b)	Attempt **any Two** of the following:	**10**
	i)	Show that if $f: \mathbb{R}^n \to \mathbb{R}^m$ is differentiable at p, then f is continuous at p.	**5**
	ii)	Calculate the Jacobian matrix for $f(x, y, z) = (x^2y, e^{yz}, z^3)$ at (1,1,1).	**5**
	iii)	State and prove the chain rule for differentiable functions on $\mathbb{R}^n$.	**5**
Q.3	**a)**	State and prove Taylor's theorem for functions of several variables.	**10**
	b)	Attempt **any Two** of the following:	**10**
	i)	Use Lagrange's method of multipliers to find the maximum value of $f(x, y) = x^2 + y^2$, subject to $x + y = 1$.	**5**
	ii)	Explain the concept of contraction mapping theorem with an example.	**5**
	iii)	Explain the second derivative test for extrema of real-valued functions on $\mathbb{R}^n$.	**5**
Q.4	**a)**	Define Riemann integration in $\mathbb{R}^n$ and state its properties.	**10**
	b)	Attempt **any Two** of the following:	**10**
	i)	Explain the concept of measure zero sets with an example.	**5**
	ii)	Evaluate $\int_0^1 \int_0^1 (x + y)\, dxdy$.	**5**
	iii)	State and prove Fubini's theorem for Riemann integration.	**5**

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